Note — Every subgph of a planar gph is planar

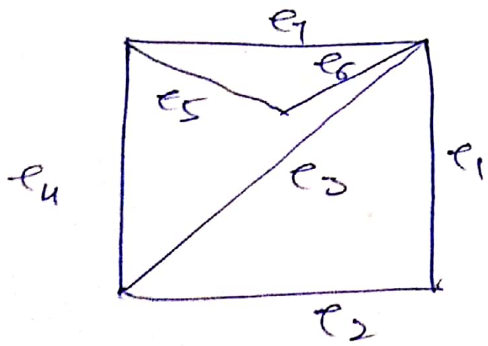
Note — Every graph which has a non planar subgph is also non planar

- A disconnected gph is planar iff each of its components is planar

Note — A separable gph (v.c. is one) is planar iff each of its blocks is planar.

- Addition or removal of self loops in a graph does not affect planarity
- // edges do not " "

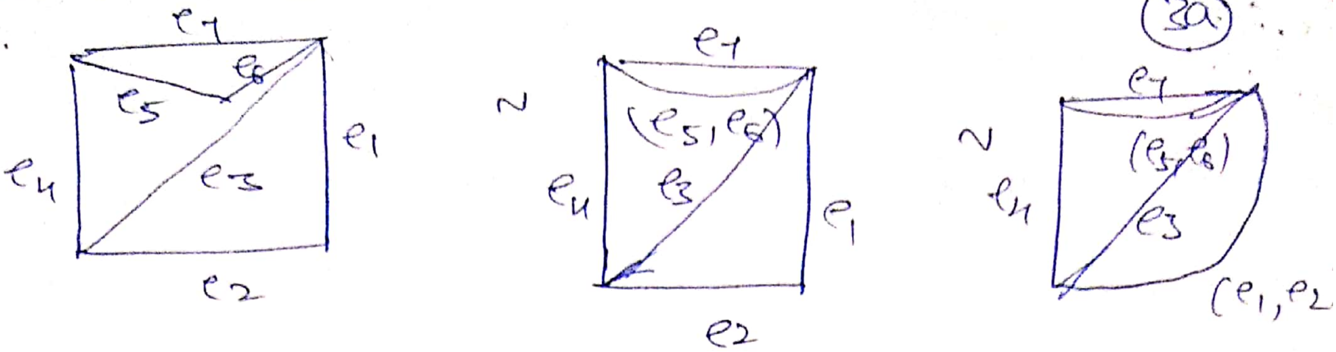
Defn → In a graph, two edges are said to be in series if they have exactly one vertex in common. and if this vertex is of degree 2. e.g.



e_5 & e_6 in series
e_1 & e_2 in series

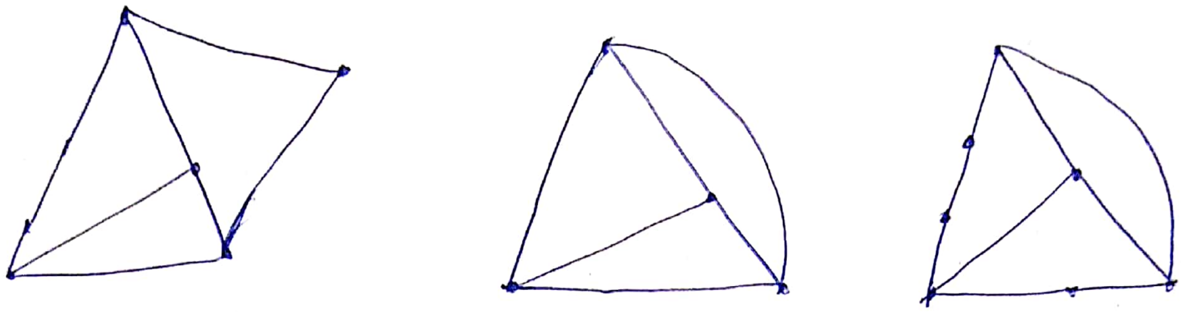
Homeomorphic Graphs → G_1 & G_2 hom. if one graph can be obtained from the other by the creation of edges in series i.e. by insertion of vertices of degree two or by the merger of edges in series.

e.g.



homeomorphic gphs.

e.g.



→ reduced edges (or vertex of deg 2)   → added edges (or vertex of deg 2)

Note → A gph is planar iff every graph that is homeomorphic to G is planar

Note A necc. & suff. condn for a gph G to be planar is that G does not contain either of Kuratowski's two gphs or any gph homeo to either of them.

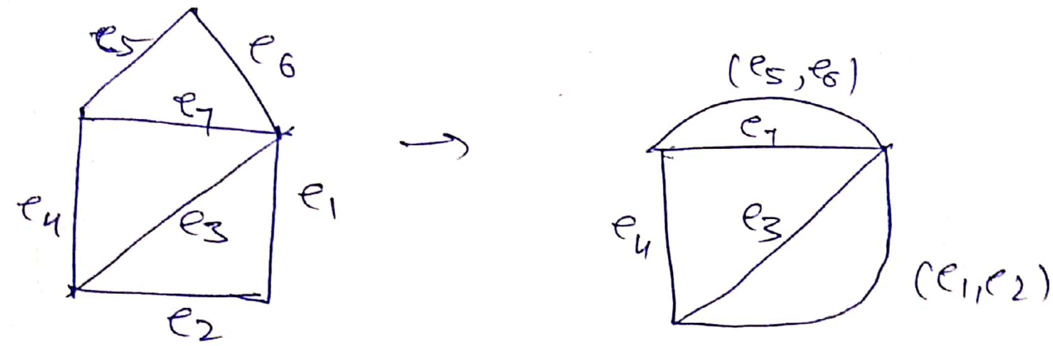
Note - A non-planar gph. may have a subgph. homeo to K's graph.

Detection of Planarity → How to find if G is planar

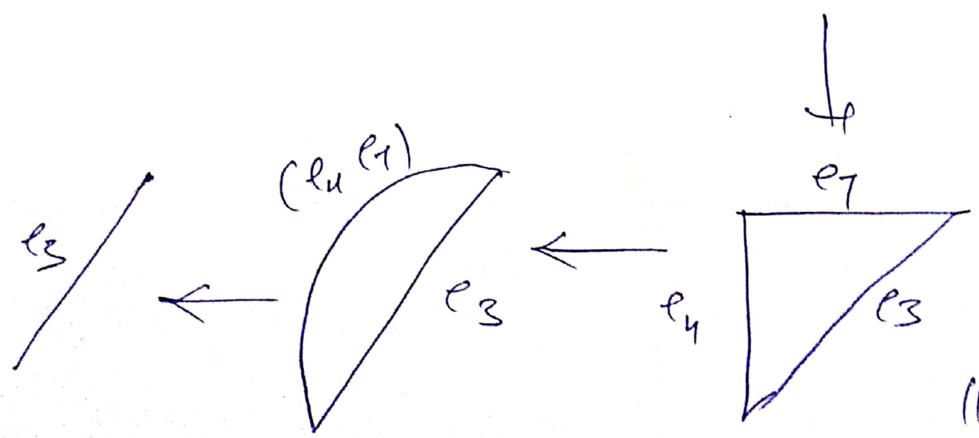
1. remove self-loops
2. " // edges
3. eliminate all edges in a series

repeat 2 & 3 we get a graph H →
This graph H is—
1. A single edge or
2. A complete graph of 4 vertices or
3. A nonseparable, simple gph. with $n \geq 5$ and $e \geq 7$

e.g.



Series reduction



// reduction

∴ we need to investigate only simple, connected non-sep. gph of at least 5 vertices and with every vertex of deg three.

Geometric Dual -
- plane representation of a gph
- place one pt. in each region.
- draw edges b/w these pts.
- if two regions are adjacent, draw a line that intersects the common edge
- more than one edge in common then draw one line b/w pts. for each of the common edges.
- for an edge lying entirely in one region, draw a self-loop.

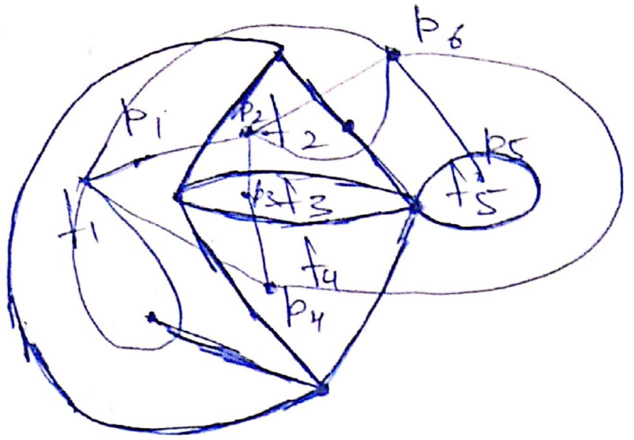
We obtain a gph. G^* → Geometric dual or Dual of G

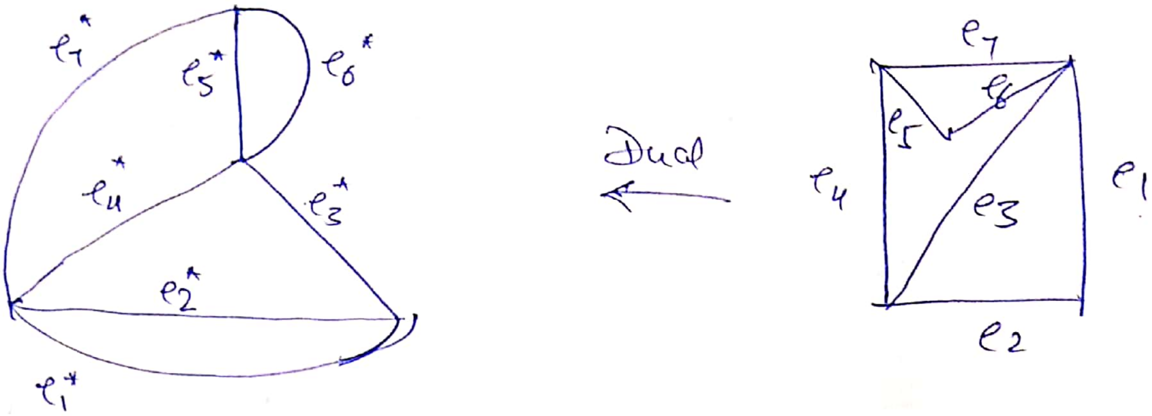
- An edge forming a self-loop in G yields a pendant edge in G^*
- A pend. edge in G yields a self-loop in G^*
- edges that are in series in G produce ∥ edges in G^*
- ∥ edges in G produce edges in series in G^*
- G^* is also planar
- G is dual of G^* & vice-versa
- n, e, f, r, μ of G

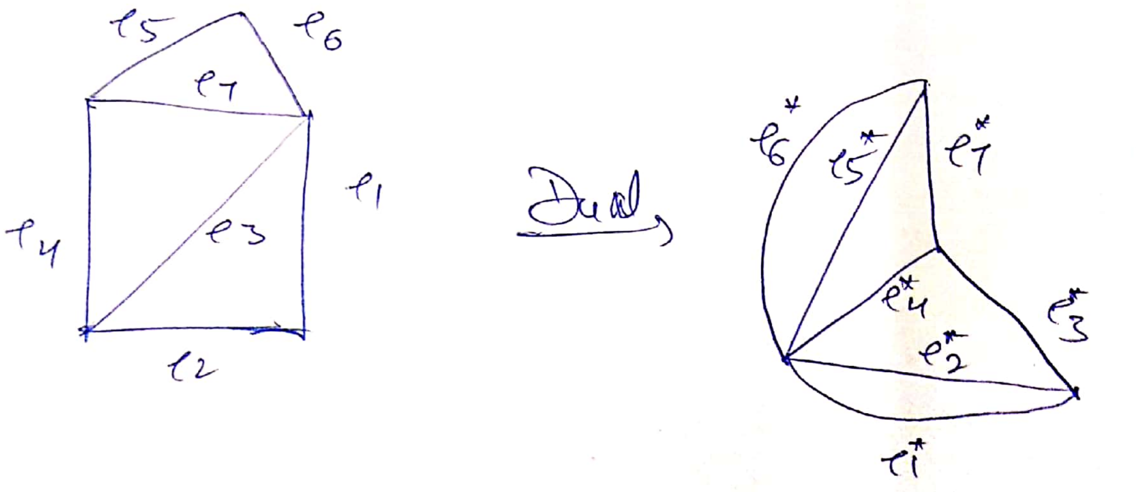
then $n^* = f$, $e^* = e$, $f^* = n$

$r^* = \mu$, $\mu^* = r$

e.g.:







Self-Dual → If a planar gph. is isomorph. to its own dual i.e. $G \cong G^*$

Complete gph. with 4 vertices is self-dual

Note → G → planar gph., G^* → Dual

Then a set of edges in G forms a circuit in G iff the corr. set of edges in G^* forms a cut-set in G^*.